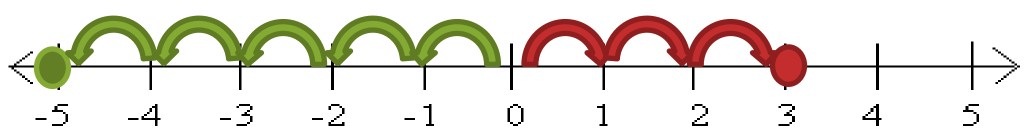
Solving Absolute Value Equations

AII.4a – The student will solve, algebraically and graphically, absolute value equations and inequalities. Graphing calculators will be used for solving and for confirming the algebraic solution.

Absolute Value

- Definition – The absolute value of a number is the DISTANCE between that number and zero on the number line.
- What do you know about distance? (Think about the odometer in a car…)
- It is always POSITIVE.
- Ex: $|3| = 3$ $|-5| = 5$

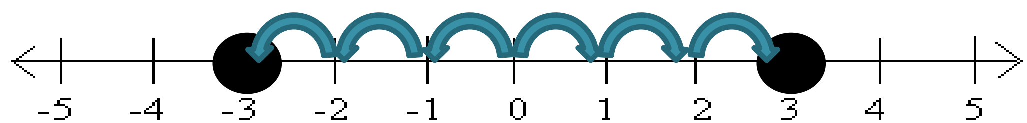


Absolute Value Equations

- If $|x| = 3$, what do you know about x?

Remember: Absolute Value is a distance.

- ***x*** has a distance of **3** from zero.
- If ***x*** is **3** 'steps' from zero on the number line, what could the value of ***x*** be?



- $x = 3$ or $x = -3$
- Thus the solution set for $|x| = 3$ is $\{x \mid x = \pm 3\}$.

Absolute Value Equations

- If $|\boldsymbol{a} + \mathbf{1}| = \mathbf{8}$, what do you know about $\boldsymbol{a} + \mathbf{1}$?
- $\boldsymbol{a} + \mathbf{1}$ is **8** steps from zero.
- If $\boldsymbol{a} + \mathbf{1}$ is **8** steps from zero, what could

Check: $|a + 1| = 8$

$\lvert 7 + 1\rvert = 8$	$\lvert -9 + 1\rvert = 8$
$\lvert 8\rvert = 8$	$\lvert -8\rvert = 8$
$8 = 8$ ✓	$8 = 8$ ✓

- Thus $a = 7$ or $a = -9$
- Be sure to always check your solutions!
- So if $|\boldsymbol{a} + \mathbf{1}| = \mathbf{8}$ then $\{\boldsymbol{a} \mid \boldsymbol{a} = \mathbf{7, -9}\}$

Absolute Value Equations

- If $|\boldsymbol{g} - \mathbf{3}| = \mathbf{10}$, what do you know about $\boldsymbol{g} - \mathbf{3}$?
- $\boldsymbol{g} - \mathbf{3}$ is **10** steps from zero. What could the va
- ***g* -**
- Th
- Solve these two equations and we get …

Check: $|g - 3| = 10$

$|13 - 3| = 10$ $\quad$ $|-7 - 3| = 10$

$|10| = 10$ $\quad$ $|-10| = 10$

$10 = 10$ ✓ $\quad$ $10 = 10$ ✓

- $g = 13$ or $g = -7$
- Be sure to always check your solutions!
- $\{\boldsymbol{g} \mid \boldsymbol{g} = \mathbf{13}, \mathbf{-7}\}$

Absolute Value Equations

- If $|5n| = -3$, what do you know about **5*n***?
- **5*n*** is **-3** steps from zero. What could the value of **5*n*** be?
- Wait, can you be **-3** steps from zero? Can distance ever be negative? NO!!
- Thus this problem has no solutions!
- We can write the solution as **Ø** or { }. It is called the null or empty set.

Absolute Value Equations

- What do you notice is different about absolute value equations when compared to other equations you have solved?

Absolute Value Equations

- What is new or different about the following equations?
- $2|x + 6| = 18$ $|4s - 8| - 7 = 3$
- Can you find the needed distance?
- No – there are extra values in the problems. What can we do?
- Use addition/subtraction/multiplication/division to get the AV expression alone on one side. NOTE – you NEVER change what is between the AV bars!!!

Absolute Value Equations

$$2|x + 6| = 18$$

Divide both sides of the equation by 2

$$|x + 6| = 9$$

Distance: *x*

9 steps fro

Solve

Check: $2|x + 6| = 18$

$2|3 + 6| = 18$	$2|-15 + 6| = 18$
$2|9| = 18$	$2|-9| = 18$
$2(9) = 18$	$2(9) = 18$
$18 = 18$ ✓	$18 = 18$ ✓

Be sure to always check your solutions!

$$\{x \mid x = 3, -15\}$$

Absolute Value Equations

$|4s - 8| - 7 = 3$

Add 7 to both sides of the equation

Distanc
10 steps

Solve

Check: $|4s - 8| - 7 = 3$

$\lvert 4(^9/_2) - 8\rvert - 7 = 3$	$\lvert 4(^{-1}/_2) - 8\rvert - 7 = 3$
$\lvert 4(^9/_2) - 8\rvert = 10$	$\lvert 4(^{-1}/_2) - 8\rvert = 10$
$\lvert 18 - 8\rvert = 10$	$\lvert -2 - 8\rvert = 10$
$\lvert 10\rvert = 10$	$\lvert 10\rvert = 10$
$10 = 10$ ✓	$10 = 10$ ✓

Be sure to always check your solutions!

$\{s \mid s = {}^9/_2, {}^{-1}/_2\}$

Absolute Value Equations

Subtract 6 from both sides of the equation

$$|3d - 9| + 6 = 0$$

$$|3d - 9| = -6$$

Distance: 3*d* - 9 is -6 steps from 0.

$$d = \{\ \} \quad \text{or} \quad d = \varnothing$$

Remember, you can walk 6 steps forward, or you can walk 6 steps backwards, but you cannot walk -6 steps. Distance is always positive and is separate from the direction you are walking.

Absolute Value Equations

What if we kept solving?

$|3d - 9| = -6$

$3d - 9 = -6$ or $3d - 9 = 6$

$3d = 3$ or $3d = 15$

$d = 1, 5$

Check: $|3d - 9| = -6$

$|3(1) - 9| = -6$ $|3(5) - 9| = -6$

$|-6| = -6$ $|6| = -6$

$6 = -6$ ✗ $6 = -6$ ✗

We would still get no solution!

Absolute Value Recap

- The absolute value of a number represents the distance a number/expression is from 0 on the number line.
- You NEVER change the AV expression inside the bars.
- You can only determine the distance when the AV expression is isolated.
- Once the AV is isolated, you can use the distance to write two equations and solve.
- The distance is always the same expression – just the positive and negative value of it.

Absolute Value Equations

- If $|2m - 3| = m + 4$, what do you know about $2m - 3$?
- $2m - 3$ is $m + 4$ steps from zero. What could the value of $2m + 3$ be?
- $2m - 3 = \pm(m + 4)$
- $2m - 3 = m + 4$ or $2m - 3 = -(m + 4)$

→ if we distribute the negative in the 2nd equation,

$$2m - 3 = -m - 4$$

Absolute Value Equations

Solve $|\mathbf{2}\boldsymbol{m} - \mathbf{3}| = \boldsymbol{m} + \mathbf{4}$

Check: $|2m - 3| = m + 4$

$\lvert 2(7) - 3\rvert = (7) + 4$	$\lvert 2(-{}^{1}/_{3}) - 3\rvert = (-{}^{1}/_{3}) + 4$
$\lvert 14 - 3\rvert = 11$	$\lvert {}^{-2}/_{3} - 3\rvert = {}^{11}/_{3}$
$\lvert 11\rvert = 11$	$\lvert {}^{-11}/_{3}\rvert = {}^{11}/_{3}$
$11 = 11$ ✓	${}^{11}/_{3} = {}^{11}/_{3}$ ✓

- $\{m \mid m = 7, -{}^{1}/_{3}\}$

Absolute Value Equations

$$|8 + 5a| = 14 - a$$

Distance: $8 + 5a$ is $14 - a$ steps from 0

$$8 + 5a = \pm(14 - a)$$

Check: $|8 + 5a| = 14 - a$

$\lvert 8 + 5(1)\rvert = 14 - 1$	$\lvert 8 + 5(-5.5)\rvert = 14 - (-5.5)$
$\lvert 13\rvert = 13$	$\lvert -19.5\rvert = 19.5$
$13 = 13$ ✓	$19.5 = 19.5$ ✓

Solve

Be sure to always check your solutions!

$$\{a \mid a = 1, {}^{-11}/_{2}\}$$

Absolute Value Equations

$$2|x| + 4 = 6x - 8$$

Isolate

divide

order n

Check: $2|x| + 4 = 6x - 8$

$2|3| + 4 = 6(3) - 8$	$2|{}^{3}/_{2}| + 4 = 6({}^{3}/_{2}) - 8$
$2(3) + 4 = 18 - 8$	$3 + 4 = 9 - 1$
$10 = 10$ ✓	$7 = 1$ ✗

Distan

steps fi

$$x = 3x - 6 \quad \text{or} \quad x = -3x + 6$$

Solve

$$x = 3, {}^{3}/_{2} \text{ (or 1.5)}$$

Be sure to always check your solutions!

Absolute Value Equations

- Wait, $^3/_2$ did not work!! Since $^3/_2$ does not solve **$2|x| + 4 = 6x - 8$**, we must throw it out of the solution set.
- $x = {}^3/_2$ is called an *extraneous solution*. We did all the steps correctly when we solved the given equation, but all the solutions we found did not work. This is why you must check all solutions every time.
- **Thus, if $2|x| + 4 = 6x - 8$, then $\{x \mid x = 3\}$.**

Absolute Value Equations

$$|3x - 1| = 1 + 3x$$

Distance: $3x - 1$ is $1 + 3x$ steps from 0

$3x - 1 = 1 + 3x$ or $3x - 1 = -1 - 3x$

Solve

$0 = 2$ or $6x = 0$

$\varnothing$ or $x = 0$

Be sure to always check your solutions!

Check: $|3x - 1| = 1 + 3x$

$|3(0) - 1| = 1 + 3(0)$

$|-1| = 1$

$1 = 1$

$$\{x \mid x = 0\}$$

How far … and from where?

In carpentry, a stud is a vertical beam used to create support in a wall. Typically studs are positioned 2 feet apart. If there is a stud 8 feet from the intersecting wall, what are the positions of the studs on either side of the pictured stud (with respect to the intersecting wall)?

6 ft. and 10 ft.

How far … and from where?

- How can create an equation that would give us this answer?
- We are looking for a value based on how far apart two things are – the DISTANCE between them…
- ABSOLUTE VALUE!!!!

How far … and from where?

$$| \qquad\qquad | = 2$$

- Absolute value tells you the distance… what is the distance in this problem?
- 2

How far … and from where?

| - | = 2

- What does the distance represent?
- The ‘difference’ of the distances between the studs.

How far … and from where?

$$| x - 8 | = 2$$

- What do we know about the studs?
- One is at 8 feet from the intersecting wall. We don't know the other.

How far … and from where?

$$| x - 8 | = 2$$

Solve ….

$$x - 8 = \pm 2$$

$$x - 8 = 2 \quad \text{or} \quad x - 8 = -2$$

$$x = 10 \quad \text{or} \quad x = 6$$

$$x = 6, 10$$

How far … and from where?

$$| x - 8 | = 2$$

We can generalize this to be:

$$|\textit{unknown} - \text{given value}| = \text{distance}$$

$$|x - 8| = 2$$

Our solutions for x are the values 2 feet from 8.

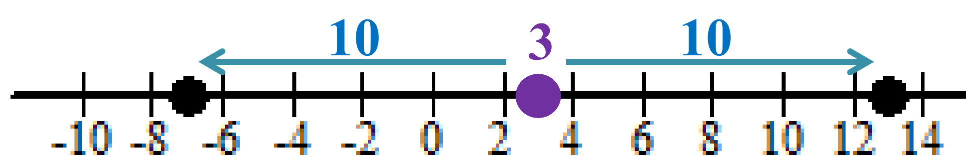
How far … and from where?

$$|\textit{unknown} - \text{given value}| = \text{distance}$$

- What if we focused on a stud 5 feet from the intersecting wall? What would be the positions of the studs beside it?
- 3 feet and 7 feet
- Create an absolute value equation that would allow us to solve for these values.
- $|x - \underline{5}| = \underline{2}$
- The solutions for x are the values 2 feet from 5.

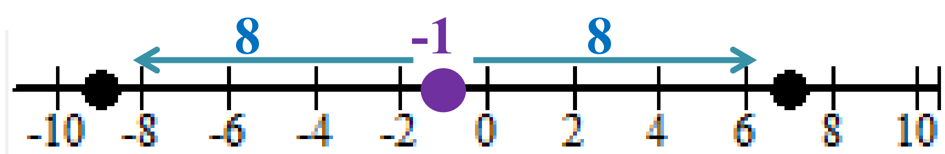
How far … and from where?

- Looking at our previous problems again…
- The solution set for **|*g* - 3| = 10** is
  {*g* | *g* = 13, -7}.
- Fill in the blanks …
  The values of *g* are __10__ steps from __3__.
- Let's prove this with the graph.

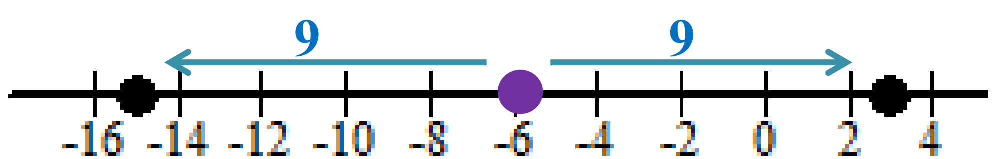


How far … and from where?

- The solution set for **$|a + 1| = 8$** is $\{a \mid a = 7, -9\}$.
- The values of a are __8__ steps from __-1__.

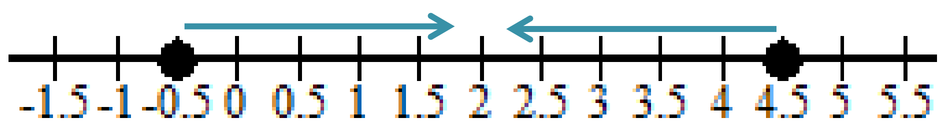


- The solution set for **$|x + 6| = 9$** (from $2|x + 6| = 18$) is $\{x \mid x = 3, -15\}$.
- The values of x are __9__ steps from __-6__.



How far … and from where?

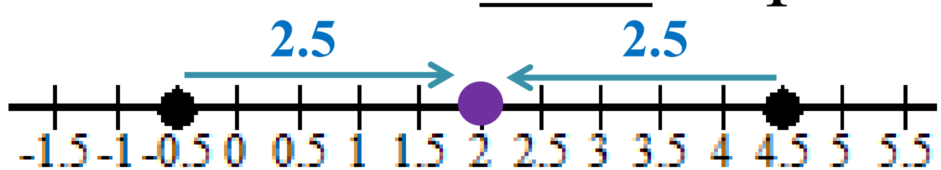
- And there is always the trouble maker…
- The solution set for **|4*s* – 8| = 10** was **{*s* | s $= {}^{9}/_{2}, {}^{-1}/_{2}$}.**
- The values of *s* are __10__ steps from __8__ right?



- **NO!!!** The two solutions are the same distance from what value?
- 2

How far … and from where?

- The solution set for $|\mathbf{4}s - \mathbf{8}| = \mathbf{10}$ was $\{s \mid s = {}^{9}/_{2}, {}^{-1}/_{2}\}$.
- The values of *s* are _2.5_ steps from _2_.



- How far are the solutions from 2?
- 2.5 steps

$|unknown - \text{given value}| = \text{distance}$

How far … and from where?

- The solution set for $|\mathbf{4}s - \mathbf{8}| = \mathbf{10}$ was $\{s \mid s = {}^{9}/_{2}, {}^{-1}/_{2}\}$.
- The values of *s* are __2.5__ steps from __2__.
- Why is this equation different?
- In the other equations, there was not a coefficient with the *unknown* variable.
- We need to keep the coefficient with the *unknown* value.

$|\textit{unknown} - \text{given value}| = \text{distance}$

How far … and from where?

- The solution set for **$|4s - 8| = 10$** was **$\{s \mid s = {}^{-1}/_{2},\ {}^{9}/_{2}\}$.**
- The values of $4s$ are __10__ steps from __8__.
- The values that are 10 steps from 8 are -2 and 18.
- So $4s = -2$ and $4s = 18$.
- Thus $s = -½$ and $s = {}^{18}/_{4} = {}^{9}/_{2}$

$|unknown - \text{given value}| = \text{distance}$

How far … and from where?

- What if our equation was $|-3f - 6| = 21$?
- The values of $-3f$ are _21_ steps from _6_.
- The values that are 21 steps from 6 are 27 and -15.
- So $-3f = 27$ and $-3f = -15$.
- Thus $f = -9$ and $f = 5$
- So the solution to $|-3f - 6| = 21$ is $\{f \mid f = -9, 5\}$

How far … and from where?

- Going back to our very first example
  $|\boldsymbol{x}| = \mathbf{3}$,

  the solutions are __3__ steps from __0__.
- How does this fit into our generalized equation?

  |*unknown* – given value| = distance
- The given value is 0 but we don't need to write the equation as $|x - 0| = 3$.

Examples

Use the generalized form of an absolute value equation to solve each problem.

- $|3m - 15| = 12$
- The values of $3m$ are __12__ steps from __15__.

$$15 \pm 12 = ?$$

$$3m = 27 \quad \text{and} \quad 3m = 3$$

$$m = 9 \quad \text{and} \quad m = 1$$

- Thus the solution set for $|3m - 15| = 12$ is $\{m \mid m = 1, 9\}$.

Examples

Use the generalized form of an absolute value equation to solve each problem.

- Solve: $\mathbf{2|5\boldsymbol{h} + 10| - 7 = 1}$
- Can we tell the distance here?
- No, we must isolate the absolute value.
- $|5h + 10| = 4$
- The values of $5h$ are __4__ steps from _-10_.
- $-10 \pm 4 = -6, -14$
- $5h = -6$ and $5h = -14$
- $\mathbf{\{\boldsymbol{h} \mid \boldsymbol{h} = -1.2, -2.8\}}$

Examples

You can still solve using straight Algebra.

- $\mathbf{2|5h + 10| - 7 = 1}$
- $|5h + 10| = 4$
- The distance is 4, so $5h + 10 = \pm 4$

$$5h + 10 = 4 \quad \text{or} \quad 5h + 10 = -4$$
$$5h = -6 \quad \text{or} \quad 5h = -14$$
$$h = -6/5 \quad \text{or} \quad h = -14/5$$
$$h = -1.2 \quad \text{or} \quad h = -2.8$$

- $\mathbf{\{h \mid h = -1.2, -2.8\}}$

One more method… graphing!

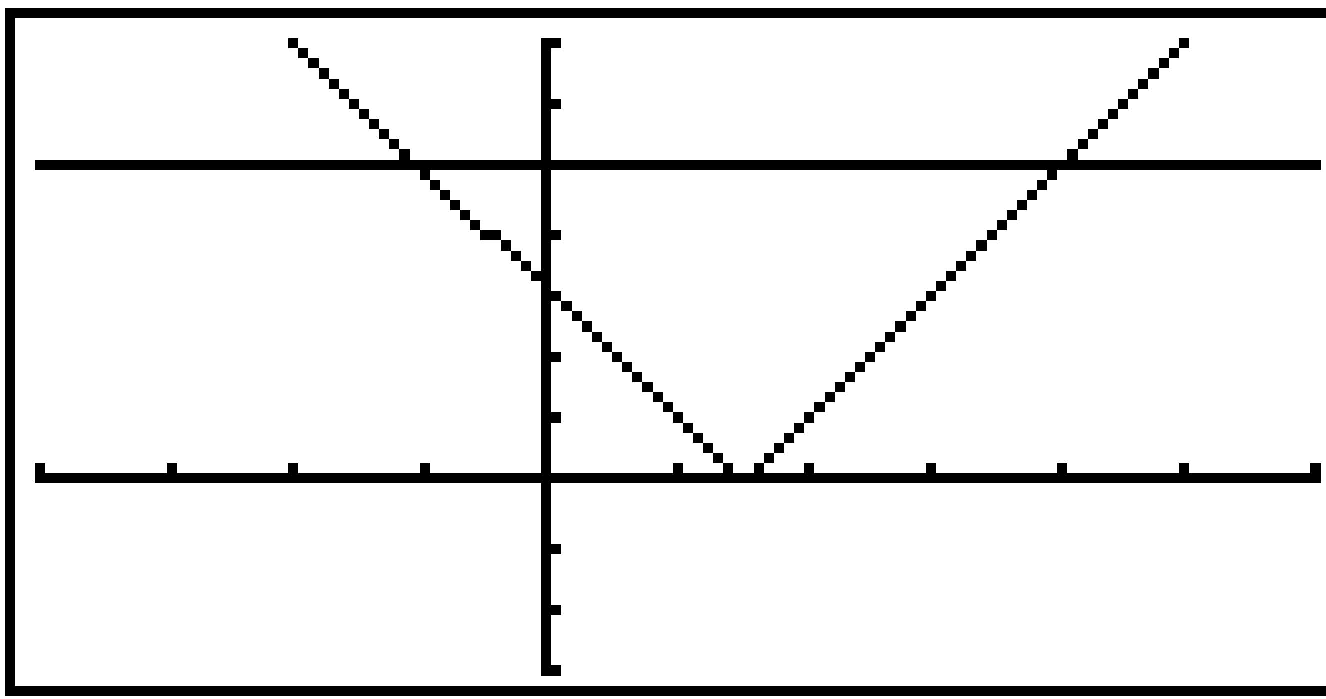
- Solve $|\mathbf{2}\boldsymbol{x}\ \mathbf{-}\ \mathbf{3}| = \mathbf{5}$

- $\{x \mid x = 4, -1\}$

- Let's explore how the graphing calculator can help us determine this solution.

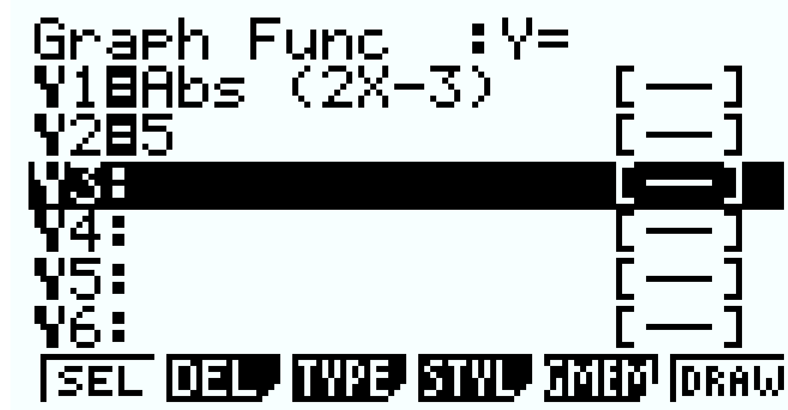
Solve by Graphing

- Graph $y = |2x - 3|$ and $y = 5$ using your graphing calculator. (See the next slides for directions.)



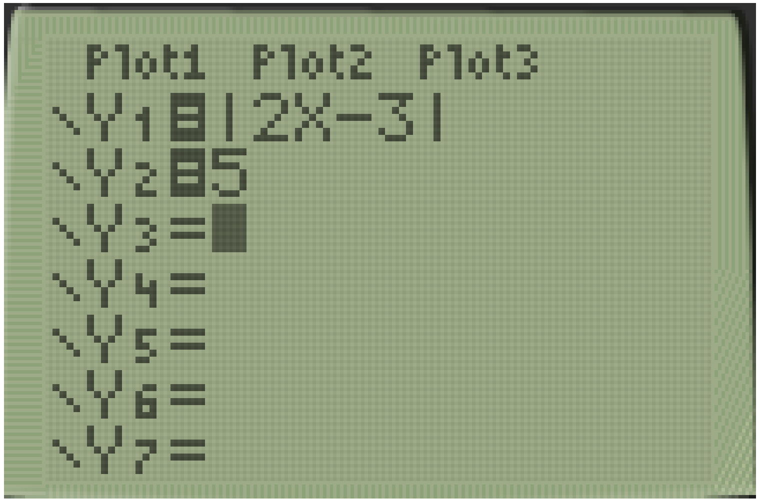
- My window: x-min: -4, x-max: 6; y-min: -3, y-max 7; all scales = 1

Graphing



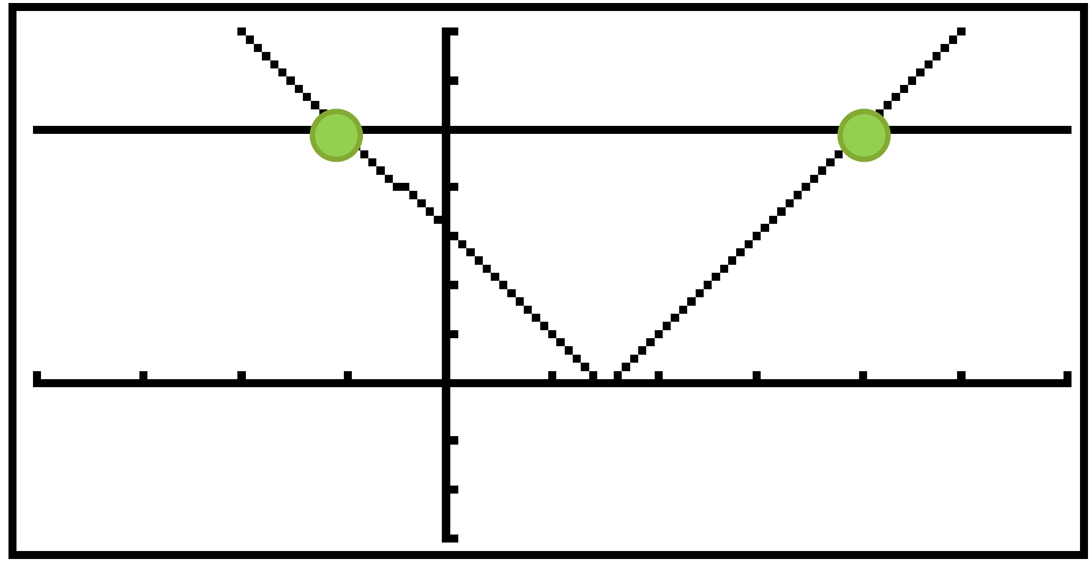
- Casio Directions
  - Main Menu – Select Graph (5)
  - Delete any equations already in the graph menu (DEL (F2), Yes (F1))
  - To find the absolute value function: hit the OPTN button. Choose NUM (F5) then Abs (F1).
  - To graph the functions hit DRAW (F6) or EXE.

Graphing



- TI Directions
  - Hit the *y* = button
  - Delete any equations already in the *y* = menu by putting the cursor on it and hitting CLEAR.
  - To find the absolute value function: hit the MATH button. Use your arrow buttons to highlight NUM then select #1: abs(.
  - Hit the GRAPH button to graph the functions.

Graphing



- Based on our graph, what could be the solutions to: $|2x - 3| = 5$?
- We want to know where the two graphs are equal, so we are looking for their intersection.
- $x = -1, 4$

Graphing

- Verify that **{*g* | *g* = 13, -7}** are solutions to **|*g* - 3| = 10** by using the graph.
- What are the two equations you need to graph?
- ***y*** **= |*g* - 3|** and ***y*** **= 10**